

Our wall as the origin of CP violation

Yutaka Sakamura [¶]

*Department of Physics, Tokyo Institute of Technology
Tokyo 152-8551, JAPAN*

Abstract

A possibility that the origin of the CP violation in our world is a domain wall itself, on which we are living, is investigated in the context of the brane world scenario. We estimate the amount of the CP violation on our wall, and show that either of order one or suppressed CP phases can be realized. An interesting case where CP is violated due to the *coexistence* of the walls, which conserve CP individually, is also considered. We also propose a useful approximation for the estimation of the CP violation in the double-wall background.

[¶]e-mail address: sakamura@th.phys.titech.ac.jp

1 Introduction

Recently, much attention is paid to topological objects such as D-branes in string theories[1], BPS domain walls[2] and junctions[3] in supersymmetric field theories. In particular, the so-called “brane world ” scenario, in which our four-dimensional world on these topological objects is embedded into a higher dimensional space-time, is investigated actively. In such scenarios, the standard model fields are confined to the brane, whereas the gravity propagates in the bulk space-time.

The authors of Refs. [4, 5] pointed out that the hierarchy problem between the Planck and weak scales may be addressed to the existence of some large extra dimensions. If such extra dimensions exist, the Planck scale M_{pl} of our world is not fundamental and related to the genuine fundamental scale M_* of a higher dimensional theory by $M_{\text{pl}}^2 \sim M_*^{2+n} V_n$, where n and V_n are a dimension and a volume of the compact extra space respectively. Thus M_* can be the TeV scale by supposing that the radii of the compactified extra dimensions are large compared to $1/M_{\text{pl}}$. However there is still a large hierarchy between M_* and $1/(V_n)^{1/n}$ in this scenario.

In contrast, a rather different idea was proposed to solve the hierarchy problem in Ref. [6]. Their model consists of our four-dimensional world and another three-brane, which are located at fixed points of an orbifold S^1/Z_2 whose radius is r_c . Assuming a non-factorizable metric of the bulk space-time, the weak scale M_w is related to the fundamental scale M_* , which is of order M_{pl} in this scenario, by $M_w = e^{-\pi k r_c} M_*$, where k is a parameter of order M_* . For the sake of the exponential dependence on the radius r_c , we can obtain large hierarchy between M_w and M_{pl} without introducing any hierarchy among the fundamental parameters. Indeed, if $k r_c \sim 12$, the observed hierarchy $M_{\text{pl}}/M_w \sim 10^{16}$ is realized.

In addition, it was pointed out in Ref. [7, 8] that the hierarchy among the fermion masses in the standard model may be explained by localizing fermions in different generations on different positions in the extra dimension. In their scenario, the fermion mass hierarchy is realized by the coupling to a Higgs condensate that falls off exponentially away from the position where the Higgs is localized.

Many other applications of the brane world scenario to cosmology and astrophysics are also investigated[9, 10].

On the other hand, the origin of the CP violation is still a mystery in particle physics. One of its candidates is the spontaneous CP violation[11]. This is convincing as a solution to the strong CP problem[12, 13] and can easily control CP violating phases appearing in the supersymmetric (SUSY) standard models or models with enlarged Higgs sector[14, 15, 16]. This scenario is based on the assumption that CP is an exact symmetry in high energy region but violated by the complex vacuum expectation values (VEVs) of some scalar fields at low energy.

In the brane world scenario, there is another possibility of the spontaneous CP violation. Namely, CP is conserved in the fundamental bulk theory but violated by the *background field configuration* instead of the complex VEVs of the scalar fields on our wall. We will investigate such a case in this paper.

Here we will assume that we are living on a four-dimensional domain wall, which interpolates two degenerate vacua of a five-dimensional bulk theory. We will call this domain wall “our wall” in this paper. In general, such a domain-wall field configuration can have a non-trivial complex phase even if parameters of the bulk theory are all real.

The purpose of this paper is to propose a mechanism of the CP violation induced by our wall itself. We investigate how the CP violation appears in the effective theory and estimate the magnitude of the typical CP phases. Then we show our mechanism can be used to realize large class of models with realistic CP violation.

In this paper, we do not discuss any gravitational effects nor gauge fields for simplicity. However, the qualitative features discussed here are supposed to be general and does not depend on the details of the theory.

The paper is organized as follows. In the next section we will explain our mechanism and give a few examples of the “complex domain-wall configurations”. Then in Section 3 we will introduce matter fields, which are trapped on our wall, and estimate the CP violation in the four-dimensional effective theory. In Section 4, another interesting model is considered. This model has two kinds of domain walls which conserve CP individually, but violate it when the two walls coexist. Section 5 is devoted to the summary. In the appendix, we introduce a particular model that has a calculable double-wall configuration, and confirm the validity of the approximation we used in Section 4.

2 CP violation due to the wall

In this section, we will illustrate our CP-violating mechanism by introducing a few simple toy models. At first, we will explain the meaning of the “complex configuration”.

Let us consider the following five-dimensional scalar theory.

$$\mathcal{L} = -\partial^M A^* \partial_M A - \Lambda^5 \left| 1 - \frac{g^n}{\Lambda^n} A^n \right|^2, \quad (M = 0, 1, \dots, 4) \quad (1)$$

where Λ is a parameter that has a dimension of the mass and g is a coupling constant whose mass-dimension is $-1/2$, and both of them are assumed to be real and positive. This model has n degenerate vacua: $A = \Lambda/g, (\Lambda/g)e^{2\pi i/n}, (\Lambda/g)e^{4\pi i/n}, \dots, (\Lambda/g)e^{2(n-1)\pi i/n}$. There is a domain-wall configuration interpolating the adjacent vacua $A = \Lambda/g$ and $(\Lambda/g)e^{2\pi i/n}$.

The most simple case is that of $n = 2$. In this case, both vacua $A = \pm\Lambda/g$ lie on the real axis of the complex target space, and the domain-wall configuration is a well-known kink solution,

$$A_{\text{cl}}(y) = \frac{\Lambda}{g} \tanh\{g\Lambda^{3/2}(y - y_0)\}, \quad (2)$$

where y is a coordinate of the fifth dimension, i.e. $y \equiv x^4$, and y_0 is a location of the domain wall. Note that this is a real function and thus has no complex phase.

When n is greater than two, however, $(\Lambda/g)e^{2\pi i/n}$ becomes a complex number and the field configuration under consideration no longer lies on the real axis, as shown in Fig.1. In other words, this background configuration has a non-trivial complex phase that depends

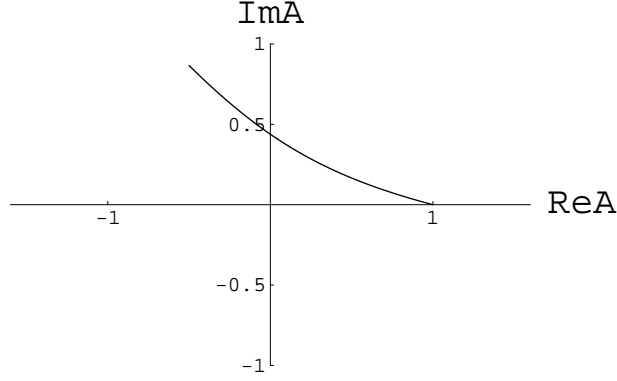


Figure 1: An example of the complex configuration. This is a domain-wall configuration interpolating the vacua: $A = 0$ and $(\Lambda/g)e^{2\pi i/3}$, of the model Eq.(1) in the case of $n = 3$. The field A is normalized by Λ/g in the figure.

on the coordinate of the fifth dimension y . We will call such a configuration a “complex configuration”.

For another example of the complex configuration, we can take a domain wall proposed in Ref. [17]. Their model is¹

$$\mathcal{L} = -\partial^M A^* \partial_M A - \frac{\Lambda^8}{|A|^2} \left| 1 - \frac{g^2}{\Lambda^2} A^2 \right|^2, \quad (3)$$

where parameters Λ and g have the same mass-dimensions as those of the previous model, and are real and positive.

This theory has two vacua $A = \pm\Lambda/g$. Though these vacua are both on the real axis, the domain wall interpolating them becomes a complex configuration due to the singularity of the potential at the origin. (See Fig.2.)

It should be noted that five-dimensional theories are non-renormalizable and have physical meanings only as effective theories of some more fundamental theories. So there is no reason for thinking highly non-renormalizable theory like Eq.(1) or Eq.(3) to be unnatural.

The y -dependent complex phases that these walls have will become the source of the CP violation in the low-energy theory.

In the following, we will assume that our wall is a complex configuration and discuss the CP violation that appear in the low-energy effective theory.

3 Effective theory and CP violation

In this section, we will introduce matter fields and investigate the zero-modes trapped on our wall, and then estimate the amount of the observed CP violation in the effective

¹ In Ref. [17], they discuss a BPS domain wall in a four-dimensional $\mathcal{N} = 1$ SUSY theory and parameters Λ and g above are set to be one.

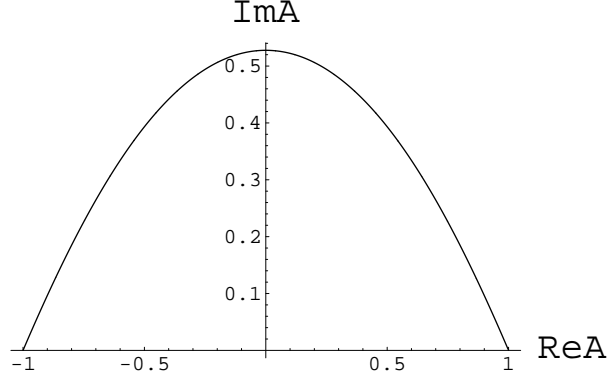


Figure 2: Another example of the complex configuration. This is a domain wall interpolating the vacua: $A = -\Lambda/g$ and Λ/g , of the model Eq.(3). The field A is normalized by Λ/g .

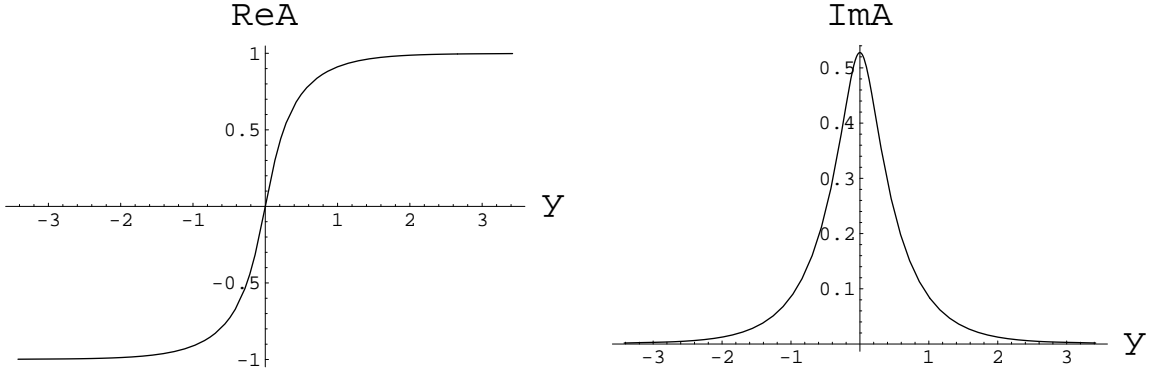


Figure 3: The profile of the domain-wall configuration $A_{\text{cl}}(y)$ of the model Eq.(3). The field A and the coordinate y are normalized by Λ/g and $1/(g^2\Lambda^2)$ respectively.

theory of those zero-modes.

3.1 Trapped zero-modes on our wall

Here we will take the domain wall of the model Eq.(3) as an example of the complex domain wall identified with our wall, and denote its field configuration as $A_{\text{cl}}(y)$. The profile of $A_{\text{cl}}(y)$ is shown in Fig.3. We emphasize that the discussion in this section is completely general and can be applied to any kind of complex configuration, as long as $\text{Re}A_{\text{cl}}(y)$ takes different values at $y = \pm\infty$.

At first, we introduce a five-dimensional matter fermion,

$$\lambda = \begin{pmatrix} \lambda_L \\ \lambda_R \end{pmatrix}, \quad (4)$$

and assume that it interacts with the wall scalar field A as

$$\mathcal{L}_{\text{int}\lambda} = -\frac{h_L}{2}A\bar{\lambda}\lambda + h.c. = -h_L(\text{Re}A)\bar{\lambda}\lambda, \quad (5)$$

where the coupling constant h_L is real positive and $\bar{\lambda}$ represents the Dirac conjugate of λ . Then the linearized equation of motion for λ is

$$-i\gamma^\mu\partial_\mu\begin{pmatrix}\lambda_L \\ \lambda_R\end{pmatrix} + \begin{pmatrix}-1 & 0 \\ 0 & 1\end{pmatrix}\partial_y\begin{pmatrix}\lambda_L \\ \lambda_R\end{pmatrix} - h_L\text{Re}A_{\text{cl}}\begin{pmatrix}\lambda_L \\ \lambda_R\end{pmatrix} = 0, \quad (6)$$

where γ^μ denotes the four-dimensional γ -matrix in the chiral representation. Here by introducing operators $\mathcal{O}_{\lambda_L} \equiv -\partial_y - h_L\text{Re}A_{\text{cl}}(y)$ and $\mathcal{O}_{\lambda_R} \equiv \partial_y - h_L\text{Re}A_{\text{cl}}(y)$, mode functions $\varphi_{\lambda_L,n}(y)$ and $\varphi_{\lambda_R,n}(y)$ are defined as solutions of the following mode equations.

$$\mathcal{O}_{\lambda_L}\varphi_{\lambda_L,n}(y) = m_n\varphi_{\lambda_R,n}(y), \quad \mathcal{O}_{\lambda_R}\varphi_{\lambda_R,n}(y) = m_n\varphi_{\lambda_L,n}(y). \quad (7)$$

Using these mode functions, the five-dimensional spinor fields can be expanded as

$$\lambda_L(x, y) = \sum_n \varphi_{\lambda_L,n}(y)\lambda_{Ln}(x), \quad \lambda_R(x, y) = \sum_n \varphi_{\lambda_R,n}(y)\lambda_{Rn}(x), \quad (8)$$

where x denotes the four-dimensional coordinates. In this case, the expansion coefficients $\lambda_{Ln}(x)$ and $\lambda_{Rn}(x)$ are regarded as the left- and right-handed components of the four-dimensional Dirac spinor fields with masses m_n [7].

For the sake of the index theorem[18], there is a zero-mode in λ_L . Its mode function $\varphi_{\lambda_L,0}(y)$ satisfies an equation $\mathcal{O}_{\lambda_L}\varphi_{\lambda_L,0}(y) = 0$,² and thus it is

$$\varphi_{\lambda_L,0}(y) = C_{\lambda_L}e^{-h_L\int_0^y dy'\text{Re}A_{\text{cl}}(y')}, \quad (9)$$

where C_{λ_L} is a normalization factor. This mode is localized on our wall ($y = 0$)[19]. Here the corresponding four-dimensional field $\lambda_{L0}(x)$ is a massless chiral fermion.

Now we will introduce another bulk fermion,

$$\chi = \begin{pmatrix}\chi_L \\ \chi_R\end{pmatrix}, \quad (10)$$

and assume an interaction with A as

$$\mathcal{L}_{\text{int}\chi} = h_R(\text{Re}A)\bar{\chi}\chi, \quad (11)$$

where h_R is real positive. In the same way, there is a zero-mode localized on our wall, and its mode function is

$$\varphi_{\chi_R,0}(y) = C_{\chi_R}e^{-h_R\int_0^y dy'\text{Re}A_{\text{cl}}(y')}, \quad (12)$$

² A solution of the equation: $\mathcal{O}_{\lambda_R}\varphi_{\lambda_R,0}(y) = 0$ is not normalizable.

where C_{χ_R} is a normalization factor. Note that the corresponding massless field $\chi_{R0}(x)$ has an opposite chirality to $\lambda_{L0}(x)$.

Next we will investigate a scalar mode trapped on our wall. At first, let us consider the fluctuation mode around the background $A_{\text{cl}}(y)$.

$$A(x, y) = A_{\text{cl}}(y) + \tilde{A}(x, y). \quad (13)$$

The linearized equation of motion for \tilde{A} is

$$\partial^M \partial_M \tilde{A} - \frac{\partial f^*}{\partial A^*} \frac{\partial f}{\partial A} \Big|_{A=A_{\text{cl}}} \tilde{A} - \frac{\partial^2 f^*}{\partial A^{*2}} f \Big|_{A=A_{\text{cl}}} \tilde{A}^* = 0, \quad (14)$$

where

$$f(A) \equiv \frac{\Lambda^4}{A} \left(1 - \frac{g^2}{\Lambda^2} A^2 \right). \quad (15)$$

If we define the operator:

$$\mathcal{O}_A \equiv -\partial_y^2 + \frac{\partial f^*}{\partial A^*} \frac{\partial f}{\partial A} \Big|_{A=A_{\text{cl}}} + \frac{\partial^2 f^*}{\partial A^{*2}} f \Big|_{A=A_{\text{cl}}}, \quad (16)$$

the mode functions $\phi_{A,n}(y)$ are defined as a solution of the equation,

$$\mathcal{O}_A \phi_{A,n}(y) = m_n^2 \phi_{A,n}(y). \quad (17)$$

Using these mode functions, we can expand the fluctuation field \tilde{A} as

$$\tilde{A}(x, y) = \sum_n \phi_{A,n}(y) a_n(x). \quad (18)$$

Here $a_n(x)$ is regarded as four-dimensional scalar field with mass m_n .

In general, there is a zero-mode in Eq.(17) corresponding to the Nambu-Goldstone mode (NG mode) for the breaking of the translational invariance along the y -direction. Its mode function is

$$\phi_{A,0}(y) = C_A \partial_y A_{\text{cl}}(y), \quad (19)$$

where C_A is a normalization factor. This mode is localized on our wall.

Finally, we will introduce a bulk complex scalar field B . To obtain a zero-mode localized on our wall, we will assume an interaction as follows.

$$\mathcal{L}_{\text{int}B} = -\frac{1}{2} \left(f^*(A^*) \frac{\partial^2 f}{\partial A^2}(A) B^2 + h.c. \right) - \left| \frac{\partial f}{\partial A}(A) B \right|^2. \quad (20)$$

Then the linearized equation of motion for B is

$$\partial^M \partial_M B - \frac{\partial f^*}{\partial A^*} \frac{\partial f}{\partial A} \Big|_{A=A_{\text{cl}}} B - \frac{\partial^2 f^*}{\partial A^{*2}} f \Big|_{A=A_{\text{cl}}} B^* = 0. \quad (21)$$

This equation is the same as that for $\tilde{A}(x, y)$ in Eq.(14), and thus there exists a zero-mode in B ³ and its mode function $\phi_{B,0}(y)$ is the same as $\phi_{A,0}(y)$, that is,

$$\phi_{B,0}(y) = C_A \partial_y A_{\text{cl}}(y). \quad (22)$$

This mode is localized on our wall.

Note that C_A is a real number in contrast to C_{λ_L} and C_{χ_R} , which are in general complex. This stems from the fact that the linearized equation of motion Eq.(21) contains both B and B^* , while that for λ or χ involves only λ or χ respectively.

3.2 Estimation of the observable CP violation

In order to discuss a four-dimensional effective theory, we will add the following interaction terms to the original Lagrangian Eq.(3).

$$\begin{aligned} \mathcal{L}_{\text{int}} = & \sum_{i=1}^{n_g} \left(M_{\lambda_i} \bar{\lambda}_i \lambda_i - h_{L_i} (\text{Re} A) \bar{\lambda}_i \lambda_i \right) + \sum_{j=1}^{n_g} \left(M_{\chi_j} \bar{\chi}_j \chi_j + h_{R_j} (\text{Re} A) \bar{\chi}_j \chi_j \right) \\ & - \frac{1}{2} \left(f^*(A^*) \frac{\partial^2 f}{\partial A^2}(A) B^2 + h.c. \right) - \left| \frac{\partial f}{\partial A}(A) B \right|^2 \\ & + \left(\sum_{i,j} y_{ij} B \bar{\chi}_j \lambda_i + h.c. \right), \end{aligned} \quad (23)$$

where $h_{L_i}, h_{R_j} > 0$ and $M_{\lambda_i}, M_{\chi_j}$ and y_{ij} are real. Here n_g denotes the number of generations. The mass terms for the fermions play a role of shifting the positions of localized zero-modes to realize the hierarchy among the Yukawa couplings[8].

There are zero-modes $\lambda_{L,i,0}(x)$, $\chi_{R,j,0}(x)$ and $b_0(x)$ in λ_i , χ_j and B respectively, which are all localized on our wall. Their mode functions are

$$\begin{aligned} \varphi_{\lambda_{L,i,0}}(y) &= C_{\lambda_{L,i}} e^{-\int_0^y dy' (h_{L,i} \text{Re} A_{\text{cl}}(y') - M_{\lambda_i})}, \\ \varphi_{\chi_{R,j,0}}(y) &= C_{\chi_{R,j}} e^{-\int_0^y dy' (h_{R,j} \text{Re} A_{\text{cl}}(y') + M_{\chi_j})}, \\ \phi_{B,0}(y) &= C_A \partial_y A_{\text{cl}}(y), \end{aligned} \quad (24)$$

where $C_{\lambda_{L,i}}, C_{\chi_{R,j}}$ are complex and C_A is real. To localize fermionic zero-modes $\lambda_{L,i,0}(x)$ on our wall, the bulk mass parameters M_{λ_i} and couplings $h_{L,i}$ must satisfy the condition that functions $f(y) \equiv h_{L,i} \text{Re} A_{\text{cl}}(y) + M_{\lambda_i}$ should take opposite sign at $y = \infty$ and $y = -\infty$ [18]. The similar condition exists for M_{χ_j} and $h_{R,j}$. We will suppose that these conditions are satisfied.

³ When we include an additional scalar field like B into the theory, we should recalculate the classical background, involving all scalar fields. In our case, however, the field configuration: $A = A_{\text{cl}}, B = 0$, is still a minimal-energy configuration (at least at the classical level) under the boundary condition: $A(y = \pm\infty) = \pm\Lambda/g, B(y = \pm\infty) = 0$, although there may be other configuration with the same energy.

By integrating out the massive modes, we can obtain a four-dimensional effective theory of massless zero-modes. The effective Yukawa couplings $y_{\text{eff}ij}$ involving $\lambda_{Li,0}$, $\chi_{Rj,0}$ and b_0 are calculated as [2, 20]

$$y_{\text{eff}ij} = y_{ij} \int_{-\infty}^{\infty} dy \phi_{B,0}(y) \varphi_{\chi_{Rj,0}}(y) \varphi_{\lambda_{Li,0}}(y). \quad (25)$$

The CP violation in this effective theory appears as the complex phases of the Yukawa couplings, i.e. $\zeta_{ij} \equiv \arg(y_{\text{eff}ij})$. However some of them can be absorbed by the field redefinition of $\lambda_{Li,0}(x)$ and $\chi_{Rj,0}(x)$. The number of the Yukawa couplings is n_g^2 and all the couplings are in general complex. On the other hand, $(2n_g - 1)$ phases can be absorbed by the field redefinition of $\lambda_{Li,0}(x)$ and $\chi_{Rj,0}(x)$. So the number of the physical phases that cannot be removed is $(n_g - 1)^2$. In particular, when we consider the case of $n_g = 3$, the following four unremovable CP phases exist.

$$\begin{aligned} \eta_1 &= \zeta_{11} + \zeta_{22} - \zeta_{12} - \zeta_{21}, \\ \eta_2 &= \zeta_{22} + \zeta_{33} - \zeta_{23} - \zeta_{32}, \\ \eta_3 &= \zeta_{33} + \zeta_{11} - \zeta_{31} - \zeta_{13}, \\ \eta_4 &= \zeta_{12} + \zeta_{23} + \zeta_{31} - \zeta_{21} - \zeta_{13} - \zeta_{32}. \end{aligned} \quad (26)$$

Of course, these phases are independent of the normalization factors of $\varphi_{\lambda_{Li,0}}(y)$ and $\varphi_{\chi_{Rj,0}}(y)$. Then we will take a quantity:

$$\Delta \equiv \max(\eta_1, \eta_2, \eta_3, \eta_4) \quad (27)$$

as a measure of the CP violation. When we try to realize the standard model in this framework, the Kobayashi-Maskawa (KM) phase is naively thought to be of order Δ .

Note that matter fermions $\lambda_{Li,0}(x)$ and $\chi_{Ri,0}$ does not interact with the NG boson $a_0(x)$ directly because only left-handed (right-handed) zero-mode exists in λ (χ). So no additional CP violating interactions are induced from the first and the second terms of Eq.(23), which play an important role of localizing the zero-modes on our wall.

It can be shown that the CP phase of order one can be realized in our mechanism. As an example, let us consider the case that

$$\begin{aligned} \frac{1}{g^3 \Lambda} (h_{L1}, h_{L2}, h_{L3}) &= \frac{1}{g^3 \Lambda} (h_{R1}, h_{R2}, h_{R3}) = (20, 12, 8), \\ -\frac{1}{g^2 \Lambda^2} (M_{\lambda 1}, M_{\lambda 2}, M_{\lambda 3}) &= \frac{1}{g^2 \Lambda^2} (M_{\chi 1}, M_{\chi 2}, M_{\chi 3}) = (-17, 0, 6). \end{aligned} \quad (28)$$

The profiles of the mode functions of fermionic modes are shown in Fig.4. In this case, the resulting CP phase is $\Delta = 0.520$. Thus we can realize the minimal standard model with the KM phase of order one.

In general, additional sources of CP violation appear if we try to extend the standard model. For example, in the supersymmetric standard models, there exist additional CP phases in the soft SUSY breaking parameters. If these phases are allowed to be of order

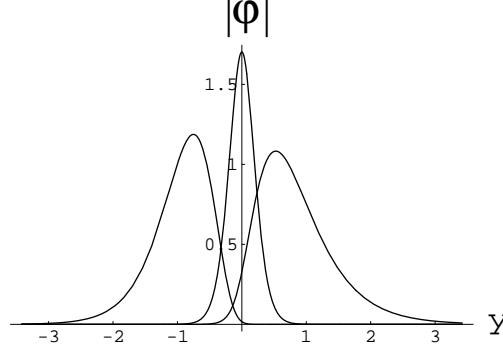


Figure 4: The profiles of the mode functions of the fermions when the parameters are set as Eq.(28). From left to right, $\varphi_{\lambda_L 1,0} = \varphi_{\chi_R 1,0}$, $\varphi_{\lambda_L 2,0} = \varphi_{\chi_R 2,0}$ and $\varphi_{\lambda_L 3,0} = \varphi_{\chi_R 3,0}$ are plotted respectively. The coordinate y is normalized by $1/(g^2\Lambda^2)$.

one, too large CP violation might occur[21]. Therefore in such a case, some mechanism is needed in order to suppress the CP phases. We claim that our CP violating mechanism can also realize small CP phases by setting the parameters: $\{h_{Li}, h_{Rj}, M_{\lambda i}, M_{\chi j}\}$, to different values. For instance, in the case that

$$\begin{aligned} \frac{1}{g^3\Lambda}(h_{L1}, h_{L2}, h_{L3}) &= \frac{1}{g^3\Lambda}(h_{R1}, h_{R2}, h_{R3}) = (8, 8, 8), \\ -\frac{1}{g^2\Lambda^2}(M_{\lambda 1}, M_{\lambda 2}, M_{\lambda 3}) &= \frac{1}{g^2\Lambda^2}(M_{\chi 1}, M_{\chi 2}, M_{\chi 3}) = (-6, 0, 6), \end{aligned} \quad (29)$$

the result is $\Delta = 3.84 \times 10^{-3}$.

There is another way of controlling the CP phases, which seems to be somewhat natural, especially when the supersymmetric extension of the models are considered. We will discuss it in the next section.

4 CP violation due to the coexistence of the walls

In this section, we will consider a two-wall system in which CP is conserved when each wall is isolated, but violated when the two walls coexist.

Let us consider the following five-dimensional theory.

$$\mathcal{L} = -\partial^M A^* \partial_M A - \Lambda^2 |A|^2 \left| 1 - \frac{g^3}{\Lambda^3} A^3 \right|^2, \quad (30)$$

where parameters Λ and g have the same mass-dimensions as those of the model Eq.(1) and Eq.(3), and again are real and positive.

This model has four degenerate vacua $A = 0, \Lambda/g, (\Lambda/g)e^{2\pi i/3}$ and $(\Lambda/g)e^{-2\pi i/3}$, shown in Fig.5.

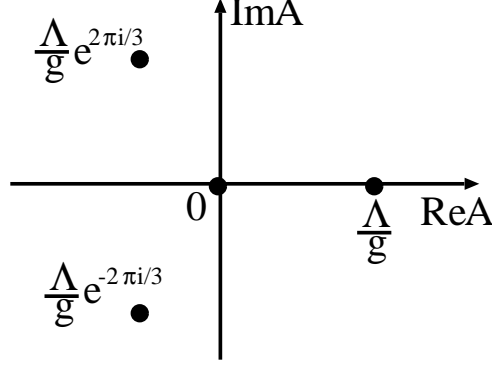


Figure 5: The degenerate vacua in the theory of Eq.(30).

There is a domain-wall configuration interpolating the vacua $A = \Lambda/g$ and $A = 0$ [22],

$$A_{\text{cl}}^{(1)}(y) = \frac{\Lambda}{g\{1 + e^{3\Lambda(y-y_1)}\}^{1/3}}. \quad (31)$$

Similarly, there is another wall configuration interpolating the vacua $A = 0$ and $A = (\Lambda/g)e^{2\pi i/3}$,

$$A_{\text{cl}}^{(2)}(y) = \frac{\Lambda e^{2\pi i/3}}{g\{1 + e^{-3\Lambda(y-y_2)}\}^{1/3}}. \quad (32)$$

Here y_1 and y_2 roughly represent the position of the walls.

These solutions have definite complex phases that is independent of y . Thus no physical CP phases are induced in the four-dimensional effective theory when we live on each of them. However if the two walls coexist at finite distance, the background configuration cannot have a definite phase any longer and has a non-trivial phase that depend on the coordinate of the extra dimension y . Therefore CP is violated in such a situation.

Now we will investigate the configuration shown in Fig.6. We assume that we live on the wall $A_{\text{cl}}^{(1)}(y)$. From now on, we will set $y_1, y_2 = 0$ in the definition of $A_{\text{cl}}^{(1)}(y), A_{\text{cl}}^{(2)}(y)$ of Eq.(31),(32).

Following Eq.(23) in the previous model, we will introduce interactions as follows,

$$\begin{aligned} \mathcal{L}_{\text{int}} = & \sum_{i=1}^{n_g} (M_{\lambda i} \bar{\lambda}_i \lambda_i + h_{Li} (\text{Re} A) \bar{\lambda}_i \lambda_i) + \sum_{j=1}^{n_g} (M_{\chi j} \bar{\chi}_j \chi_j - h_{Rj} (\text{Re} A) \bar{\chi}_j \chi_j) \\ & - \frac{1}{2} \left(f^*(A^*) \frac{\partial^2 f}{\partial A^2}(A) B^2 + h.c. \right) - \left| \frac{\partial f}{\partial A}(A) B \right|^2 \\ & + \left(\sum_{i,j} y_{ij} B \bar{\chi}_j \lambda_i + h.c. \right), \end{aligned} \quad (33)$$

where

$$f(A) \equiv \Lambda A \left(1 - \frac{g^3}{\Lambda^3} A^3 \right), \quad (34)$$

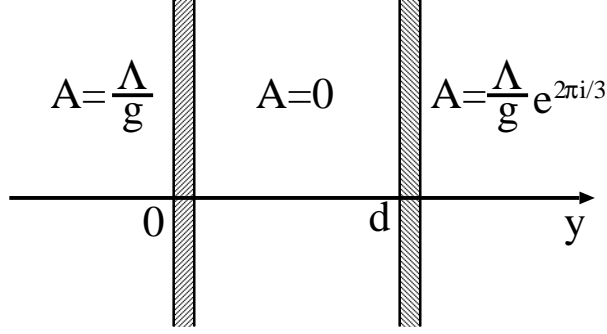


Figure 6: An interesting double-wall configuration. The wall at $y = 0$ is $A_{\text{cl}}^{(1)}(y)$ in Eq.(31) and the wall at $y = d$ is $A_{\text{cl}}^{(2)}(y)$ in Eq.(32).

and $h_{Li}, h_{Rj} > 0$ and y_{ij} are real. As in the previous model,

$$\lambda_i = \begin{pmatrix} \lambda_{Li} \\ \lambda_{Ri} \end{pmatrix}, \quad \chi_j = \begin{pmatrix} \chi_{Lj} \\ \chi_{Rj} \end{pmatrix}$$

are five-dimensional Dirac spinor fields and B is a complex scalar field.

Unfortunately, we do not know the exact double-wall configuration shown in Fig.6. However, since the field-configuration of our wall is deformed from the real configuration $A_{\text{cl}}^{(1)}(y)$ to a complex configuration by the other wall, the imaginary part of our-wall configuration can be thought to come from that of $A_{\text{cl}}^{(2)}(y-d)$, where d is the distance between the walls. Thus the background configuration $A_{\text{cl}}(y)$ shown in Fig.6 is approximated near our wall by

$$A_{\text{cl}}(y) \simeq A_{\text{cl}}^{(1)}(y) + A_{\text{cl}}^{(2)}(y-d) \simeq A_{\text{cl}}^{(1)}(y) + i\text{Im}A_{\text{cl}}^{(2)}(y-d). \quad (35)$$

The validity of this approximation is confirmed in the appendix.

Then the (pseudo) zero-modes trapped on our wall can be approximated near our wall as follows.

$$\begin{aligned} \varphi_{\lambda_{Li}}(y) &= C_{\lambda_{Li}} e^{\int_0^y dy' \{h_{Li} A_{\text{cl}}^{(1)}(y') + M_{\lambda_i}\}}, \\ \varphi_{\chi_{Rj}}(y) &= C_{\chi_{Rj}} e^{\int_0^y dy' \{h_{Rj} A_{\text{cl}}^{(1)}(y') - M_{\chi_j}\}}, \\ \phi_B(y) &= C_B \partial_y \left(A_{\text{cl}}^{(1)}(y) + i\text{Im}A_{\text{cl}}^{(2)}(y-d) \right), \end{aligned} \quad (36)$$

where $C_{\lambda_{Li}}, C_{\chi_{Rj}}$ are complex and C_B is real.

Using these mode functions, effective Yukawa couplings $y_{\text{eff}ij}$ that appear in the four-dimensional effective theory are calculated as

$$y_{\text{eff}ij} = y_{ij} \int_{-\infty}^{\infty} dy \phi_B(y) \varphi_{\chi_{Rj}}(y) \varphi_{\lambda_{Li}}(y). \quad (37)$$

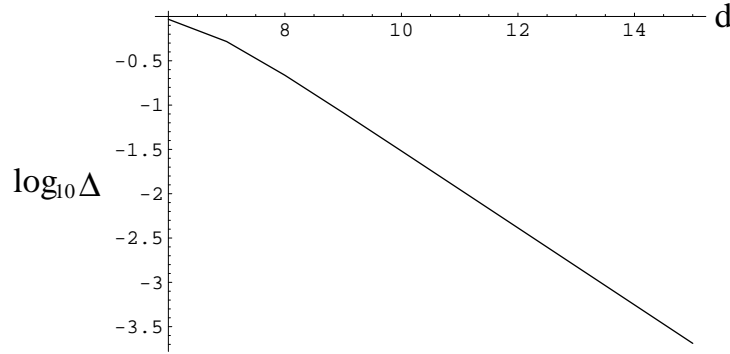


Figure 7: The measure of the CP violation Δ as a function of the wall distance in the case of Eq.(38). The distance d is normalized by $1/\Lambda$.

The measure of the CP violation Δ , defined in Eq.(27), is calculated from these $y_{\text{eff}ij}$ and it is shown in Fig.7 in the case that

$$\begin{aligned} \frac{1}{g}(h_{L1}, h_{L2}, h_{L3}) &= \frac{1}{g}(h_{R1}, h_{R2}, h_{R3}) = (20, 12, 8), \\ \frac{1}{\Lambda}(M_{\lambda1}, M_{\lambda2}, M_{\lambda3}) &= -\frac{1}{\Lambda}(M_{\chi1}, M_{\chi2}, M_{\chi3}) = (-1, -4, -7). \end{aligned} \quad (38)$$

As Fig.7 shows, this model can realize a wide range of the magnitude of Δ , and can easily control it by changing only one parameter, i.e. the distance between the walls.

When we try to extend the standard model to supersymmetric, we should specify the mechanism of the SUSY breaking. Recently, the author has proposed a new SUSY breaking mechanism with collaborators that SUSY is broken due to the coexistence of two different kinds of BPS domain walls[20]. In this mechanism, we have introduced the other wall in addition to our wall, which breaks the supersymmetry preserved by our wall. Thus if we apply this SUSY breaking mechanism with our CP violating scenario, there is a possibility that the other wall is a source of not only SUSY breaking but also CP violation. In such a case, additional CP phases appearing in the soft SUSY breaking parameters can naturally be suppressed by the wall distance just in the same way as those of the Yukawa couplings calculated above.

Note that in the localization mechanism we used so far, the mode functions of the fermionic modes cannot have non-trivial complex phases depending on y . However, this is not an inevitable feature of our scenario. Indeed, we can make the mode functions of fermions have non-trivial phases by making the first and the second terms in Eq.(33) be non-diagonal. To illustrate this, let us consider the two-generation case.

$$\mathcal{L}_{\text{int}} = (\bar{\lambda}_1, \bar{\lambda}_2) \begin{pmatrix} h_{L11}\text{Re}A + M_{\lambda11} & h_{L12}A + M_{\lambda12} \\ h_{L12}A^* + M_{\lambda12} & h_{L22}\text{Re}A + M_{\lambda22} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}, \quad (39)$$

where h_{Lij} and $M_{\lambda ij}$ are real.

We define the functions:

$$M_{Lij}(y) \equiv -(h_{Lij}A_{cl}(y) + M_{\lambda ij}) \quad (i, j = 1, 2) \quad (40)$$

Then the linearized equation of motion for λ_i is

$$-i\gamma^M \partial_M \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} - \begin{pmatrix} \text{Re}M_{L11} & M_{L12} \\ M_{L12}^* & \text{Re}M_{L22} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0. \quad (41)$$

Defining the operators:

$$\mathcal{O}_{\lambda_L} \equiv -\partial_y - \begin{pmatrix} \text{Re}M_{L11} & M_{L12} \\ M_{L12}^* & \text{Re}M_{L22} \end{pmatrix}, \quad \mathcal{O}_{\lambda_R} \equiv \partial_y - \begin{pmatrix} \text{Re}M_{L11} & M_{L12} \\ M_{L12}^* & \text{Re}M_{L22} \end{pmatrix}, \quad (42)$$

the mode functions are defined as solutions of the following equations.

$$\mathcal{O}_{\lambda_L} \begin{pmatrix} \varphi_{L1,n}(y) \\ \varphi_{L2,n}(y) \end{pmatrix} = m_n \begin{pmatrix} \varphi_{R1,n}(y) \\ \varphi_{R2,n}(y) \end{pmatrix}, \quad \mathcal{O}_{\lambda_R} \begin{pmatrix} \varphi_{R1,n}(y) \\ \varphi_{R2,n}(y) \end{pmatrix} = m_n \begin{pmatrix} \varphi_{L1,n}(y) \\ \varphi_{L2,n}(y) \end{pmatrix}. \quad (43)$$

The bulk fermion fields can be expanded by these mode functions.

$$\begin{pmatrix} \lambda_{L1}(x, y) \\ \lambda_{L2}(x, y) \end{pmatrix} = \sum_n \lambda_{L,n}(x) \begin{pmatrix} \varphi_{L1,n}(y) \\ \varphi_{L2,n}(y) \end{pmatrix}, \quad \begin{pmatrix} \lambda_{R1}(x, y) \\ \lambda_{R2}(x, y) \end{pmatrix} = \sum_n \lambda_{R,n}(x) \begin{pmatrix} \varphi_{R1,n}(y) \\ \varphi_{R2,n}(y) \end{pmatrix}. \quad (44)$$

Thus the equation for the zero-modes of \mathcal{O}_{λ_L} is

$$\partial_y \begin{pmatrix} \varphi_{L1,0}(y) \\ \varphi_{L2,0}(y) \end{pmatrix} = - \begin{pmatrix} \text{Re}M_{L11}(y) & M_{L12}(y) \\ M_{L12}^*(y) & \text{Re}M_{L22}(y) \end{pmatrix} \begin{pmatrix} \varphi_{L1,0}(y) \\ \varphi_{L2,0}(y) \end{pmatrix}. \quad (45)$$

Here we denote the asymptotic values of $M_{Lij}(y)$ as

$$M_{\pm\infty ij} \equiv \lim_{y \rightarrow \pm\infty} M_{Lij}(y). \quad (46)$$

Then the condition for two zero-modes to exist, that is, for both solutions of Eq.(45) to decay to zero at infinity, is that the eigenvalues of the following two matrices are all positive.

$$\begin{pmatrix} \text{Re}M_{\infty 11} & M_{\infty 12} \\ M_{\infty 12}^* & \text{Re}M_{\infty 22} \end{pmatrix}, \quad - \begin{pmatrix} \text{Re}M_{-\infty 11} & M_{-\infty 12} \\ M_{-\infty 12}^* & \text{Re}M_{-\infty 22} \end{pmatrix}, \quad (47)$$

In other words,

$$\begin{cases} \text{Re}M_{\infty 11} > 0 \\ (\text{Re}M_{\infty 11})(\text{Re}M_{\infty 22}) - |M_{\infty 12}|^2 > 0 \end{cases}, \quad (48)$$

and

$$\begin{cases} \text{Re}M_{-\infty 11} < 0 \\ (\text{Re}M_{-\infty 11})(\text{Re}M_{-\infty 22}) - |M_{-\infty 12}|^2 > 0 \end{cases}. \quad (49)$$

If this condition is satisfied, two normalizable zero-modes exist and their asymptotic behaviors are

$$\begin{aligned} \begin{pmatrix} \varphi_{L1,0}(y) \\ \varphi_{L2,0}(y) \end{pmatrix} &\rightarrow \exp \left\{ - \begin{pmatrix} \text{Re} M_{\infty 11} & M_{\infty 12} \\ M_{\infty 12}^* & \text{Re} M_{\infty 22} \end{pmatrix} y \right\} \begin{pmatrix} C_{1,\infty} \\ C_{2,\infty} \end{pmatrix}, \quad (y \rightarrow \infty) \\ &\rightarrow \exp \left\{ - \begin{pmatrix} \text{Re} M_{-\infty 11} & M_{-\infty 12} \\ M_{-\infty 12}^* & \text{Re} M_{-\infty 22} \end{pmatrix} y \right\} \begin{pmatrix} C_{1,-\infty} \\ C_{2,-\infty} \end{pmatrix}, \quad (y \rightarrow -\infty) \end{aligned} \quad (50)$$

where $C_{i,\pm\infty}$ ($i = 1, 2$) are complex constants.

Unlike the previous case, these mode functions can have non-trivial complex phases. The discussion for the modes in χ_j is the same.

The extension to the three or more generation cases is straightforward.

5 Summary

We discussed the origin of the CP violation in the context of the brane world scenario, especially in the case that the three brane where we live is a domain wall in a five-dimensional space-time. In such a case, there is a new possibility that the origin of the CP violation in our world is the domain wall itself.

In the large class of models, even if the bulk theory is CP-invariant, it has a domain-wall configuration with a non-trivial complex phase that depends on the coordinate of the extra dimension y . Then the mode functions of the trapped modes on our wall become complex functions. Since effective couplings in the four-dimensional effective theory are obtained by overlap integrals of these complex mode functions, non-zero CP phases appear in the effective theory. These CP phases cannot be removed by field redefinition because of the non-trivial y -dependence of the background configuration. As a result, CP violation occurs in the effective theory.

This mechanism can be classified into the spontaneous CP violation, but it violates the CP-invariance by the “complex wall-configuration”, instead of the complex VEVs of some scalar fields.

In the domain-wall scenario like ours, the hierarchy among the Yukawa couplings can easily be realized by locating the fermions at different positions in our wall[8]. We showed that our scenario can give $O(1)$ CP phases by using this mechanism together. So we can construct the minimal standard model in our scenario.

When we extend the standard model, additional sources of the CP violation come out and they often need to be suppressed to avoid the contradiction to the experimental data. Our mechanism can also be applied to such a case because we can realize the small CP phases, too. Especially we considered an interesting case in which the CP violation in our world is caused by the existence of the other wall, which is located distant from our wall along the extra dimension. When our wall or the other wall is isolated, each wall configuration has a definite complex phase independent of y , and thus CP is not violated in the effective theory on each wall. However, when the two walls coexist at finite distance, the background configuration no longer has a definite phase and the CP violation occurs.

on our wall. We emphasize that our CP violating mechanism does not need any bulk fields mediating the CP violating effects to our wall or any source of the CP violation on the other wall, in contrast to Ref. [23]. CP is violated only by the “coexistence of the walls”.

This double-wall scenario is congenial to the SUSY breaking scenario proposed in Ref. [20], in which SUSY is broken due to the coexistence of two walls. Namely we can consider an attractive scenario that the CP violation and the SUSY breaking have the common origin, that is, the coexistence of our wall and the other wall.

One of the characteristic features of our double-wall scenario is that the CP violation observed on our wall decays exponentially as the distance between the walls increases.

We also proposed a practical method for estimation of the CP violation induced on our wall in the double-wall scenario. We often encounter the case that only a single-wall configuration is known exactly but we do not know an exact double-wall configuration representing the coexistence of our wall and the other wall. We showed that even in such a case, the estimation of the CP violation is possible by an appropriate approximation using only a knowledge about the single-wall configuration. The validity of this approximation is also confirmed in the appendix.

Finally, We emphasize that our CP violating mechanism has a flexibility to realize both types of CP violating models: models with an $O(1)$ CP phase such as the Kobayashi-Maskawa model and models with small CP phases such as the supersymmetric standard models. So the possibility discussed in this paper should be taken into account in the model-building of a realistic model in the context of the domain-wall scenario.

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A Calculable double-wall configuration

Here we will consider a particular model that has a calculable double-wall configuration, and confirm the validity of the approximation Eq.(35).

A.1 Classical background

We will introduce a five-dimensional theory as follows.

$$\mathcal{L} = -\partial^M A^* \partial_M A - \Lambda^5 \left| \frac{\frac{1}{2} - \cos(\frac{g}{\Lambda} A + \alpha)}{2 - \cos \frac{g}{\Lambda} A} \right|^2, \quad (M = 0, 1, \dots, 4) \quad (51)$$

where α is a real parameter, and parameters Λ and g have the same mass-dimensions as those of the model Eq.(1) or Eq.(3) and are positive. Here, the fifth dimension is compactified on S^1 whose radius is R , and its coordinate is denoted as y , i.e. $y \equiv x^4$. For convenience, we will redefine A and x^M as

$$\frac{g}{\Lambda} A \rightarrow A, \quad g \Lambda^{3/2} x^M \rightarrow x^M, \quad (52)$$

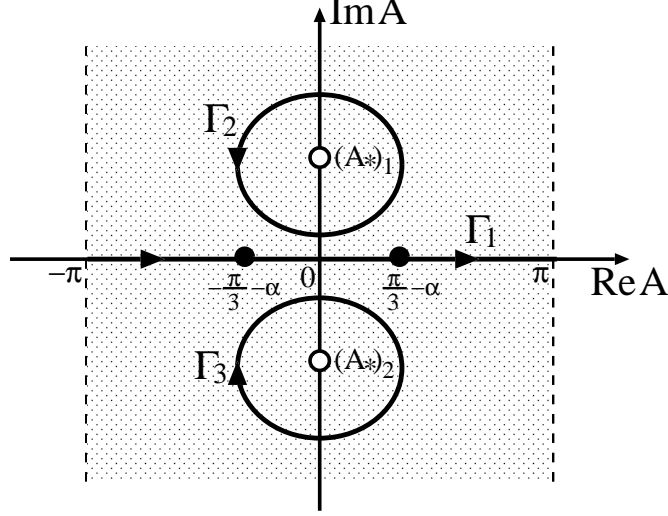


Figure 8: The target space of the scalar field A . The lines $\text{Re}A = \pi$ and $\text{Re}A = -\pi$ represent the same line.

so that A and x^M become dimensionless variables. Then at the classical level, the above theory is equivalent to the following one.

$$\mathcal{L} = -\partial^M A^* \partial_M A - \left| \frac{\frac{1}{2} - \cos(A + \alpha)}{2 - \cos A} \right|^2. \quad (53)$$

The target space of the scalar field A has a topology of a cylinder with two points $(A_*)_{1,2}$ deleted⁴ (Fig.8),

$$-\pi \leq \text{Re}A \leq \pi, \quad -\infty < \text{Im}A < \infty, \quad (54)$$

and

$$(A_*)_{1,2} = \pm i \ln(2 + \sqrt{3}). \quad (55)$$

The model has two vacua at $A = \pm\pi/3 - \alpha$ and the potential has two poles at $A = (A_*)_{1,2}$. There are three noncontractible cycles, Γ_1, Γ_2 and Γ_3 depicted in Fig.8.

Now let us consider a vacuum configuration that depends only on y . We will seek a domain-wall configuration that winds around the pole $(A_*)_1$ counterclockwise as y increases. Its trajectory on the target space corresponds to the cycle Γ_2 . Such a configuration is topologically stable. To obtain such a configuration, we will dimensionally reduce our model to the four-dimensional theory in terms of, for example, x^3 -direction. Then the problem is reduced to seeking a domain-wall configuration in the four-dimensional theory.

In this case, our model Eq.(53) can be regarded as a bosonic part of a four-dimensional $\mathcal{N} = 1$ supersymmetric model,

$$\mathcal{L}^{(4)} = \bar{\Phi} \Phi|_{\theta^2 \bar{\theta}^2} + W(\Phi)|_{\theta^2} + h.c., \quad (56)$$

⁴ This is similar to the one in Ref. [24].

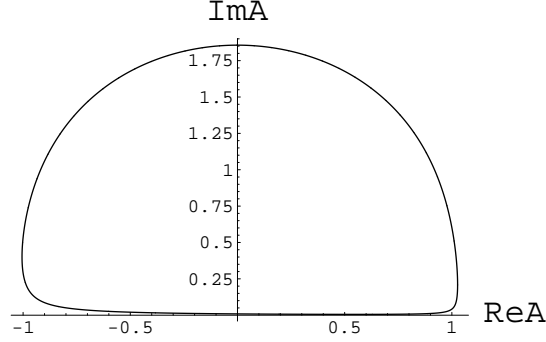


Figure 9: the BPS trajectory homotopical to the cycle Γ_2 . This is the contour corresponding to $\alpha = 0.01$ ($I_\infty = -0.00357$) and $I_0 = -0.00448$.

where $\Phi = A + \sqrt{2}\theta\psi + \theta^2 F$ is a chiral superfield, and $W(\Phi)$ is a superpotential as follows.

$$W(\Phi) = \cos \alpha \cdot \Phi + \left(\frac{1}{2} - 2 \cos \alpha\right) \frac{2}{\sqrt{3}} \tan^{-1}(\sqrt{3} \tan \frac{\Phi}{2}) + \sin \alpha \cdot \ln(2 - \cos \Phi). \quad (57)$$

We will seek a BPS configuration in this model. Such a configuration can be found by using a method proposed in Ref. [24].

The period corresponding to the cycle Γ_2 is

$$\begin{aligned} \Delta W &= \oint_{\Gamma_2} \frac{\partial W}{\partial A} dA = \oint_{\Gamma_2} \frac{\frac{1}{2} - \cos(A + \alpha)}{2 - \cos A} dA \\ &= \frac{2\pi}{\sqrt{3}} \left\{ 2 \cos \alpha - \frac{1}{2} - i\sqrt{3} \sin \alpha \right\} \end{aligned} \quad (58)$$

Thus the BPS equations are

$$\frac{dA}{dy} = e^{i\delta} \frac{\partial W^*}{\partial A^*}, \quad \frac{dA^*}{dy} = e^{-i\delta} \frac{\partial W}{\partial A}, \quad (59)$$

where $\delta \equiv \arg(\Delta W)$.

Here we will introduce the multivalued function $I(A, A^*)$, which is defined as

$$I = \text{Im}(e^{-i\delta} W). \quad (60)$$

This is the integral of motion of Eq.(59). Note that a trajectory of a BPS configuration on the field space is a contour line of $I(A, A^*) = I_0$ where I_0 is a real constant. Here we are interested in the field configuration that has a wall structure, so we will consider a contour line that passes near the two vacua $A = \pm\pi/3 - \alpha$ as shown in Fig.9. It can be obtained by putting I_0 close to the value $I_\infty \equiv \text{Im}(e^{-i\delta} W(\pi/3 - |\alpha|))$ from below.

To parametrize the contour shown in Fig.9, we will introduce $\theta \equiv \arg\{i(A - (A_*)_1)\}$ as a parameter, where A is a point on the contour. The relation between θ and y is obtained from Eq.(59),

$$y(\theta) = \int_0^\theta d\theta \frac{dA}{d\theta} e^{-i\delta} \frac{2 - \cos A^*}{\frac{1}{2} - \cos(A^* + \alpha)}. \quad (61)$$

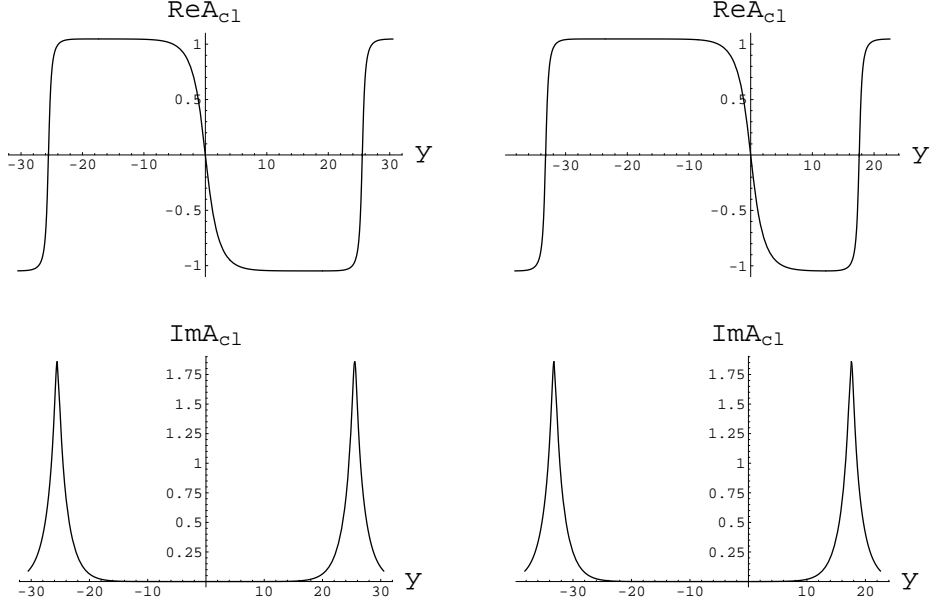


Figure 10: The profile of the classical configuration $A_{\text{cl}}(y)$ in the case of $\alpha = 0$ (the left plots) and $\alpha \neq 0$ (the right plots).

Here we set the initial condition as $\theta = 0$ at $y = 0$. By using this relation, we can obtain the classical solution $A_{\text{cl}}(y)$ for each value of I_0 .

At first, let us consider the case of $\alpha = 0$. In this case, $I_\infty = 0$ and the configuration becomes two BPS domain walls. The distance between them goes to infinity in the limit of $I_0 \rightarrow I_\infty$. These two domain walls preserve the same supersymmetry in contrast to the case discussed in Ref. [20]. This configuration has an equidistant-wall structure shown by the left plots in Fig.10. Here the period of the configuration is set to be $2\pi R$ in order to realize a double-wall system.

The wall located at $y = 0$ becomes a real configuration $A_{\text{cl}}^{(1)}(y)$ in the limit of $R \rightarrow \infty$, and we will regard it as our wall in the following. The inverse function of $A_{\text{cl}}^{(1)}(y)$ can be calculated analytically as

$$y = A_{\text{cl}}^{(1)} - 2\sqrt{3} \tanh^{-1} \left(\sqrt{3} \tan \frac{A_{\text{cl}}^{(1)}}{2} \right). \quad (62)$$

On the other hand, the other wall located at $y = \pi R$ is a complex configuration even if R goes to infinity. We will denote this wall in the limit of $R \rightarrow \infty$ as $A_{\text{cl}}^{(2)}(y)$.

Next we will consider the case of $\alpha \neq 0$. In this case, when I_0 is put close to I_∞ , the contour approaches the two vacua in an asymmetric manner and thus the configuration has a non-equidistant-wall structure shown by the right plots in Fig.10, in contrast to the cases in Ref. [17, 24]. Unlike the previous case, our wall does not become a real configuration even in the limit of $R \rightarrow \infty$ (i.e., $I_0 \rightarrow I_\infty$) and becomes a structure such that two BPS domain walls are finitely separated in a non-compact space. This configuration is similar to the one in Ref. [25].

From these facts, for a given compactified radius R , the distance between our wall and the other wall can be set to an arbitrary value by adjusting the constant I_0 and the parameter α ⁵. For simplicity, however, we will limit ourselves to the case of $\alpha = 0$ in the following discussion.

Although the classical configuration $A_{\text{cl}}(y)$ is obtained in the four-dimensional $\mathcal{N} = 1$ supersymmetric model, this configuration can be regarded as a domain wall in the five-dimensional non-supersymmetric theory of Eq.(53), because all we used in the above derivation is a bosonic part of the theory. Thus in the following, we will regard $A_{\text{cl}}(y)$ as a desired classical double-wall configuration of the model of Eq.(53).

A.2 Estimation of CP violation

We will follow the procedure in Section 3.2 to estimate the CP phase in the four-dimensional effective theory. At first, we will add the following interaction terms to the original Lagrangian Eq.(53),

$$\begin{aligned} \mathcal{L}_{\text{int}} = & \sum_{i=1}^{n_g} \left(M_{\lambda i} \bar{\lambda}_i \lambda_i + h_{Li} (\text{Re} A) \bar{\lambda}_i \lambda_i \right) + \sum_{j=1}^{n_g} \left(M_{\chi j} \bar{\chi}_j \chi_j - h_{Rj} (\text{Re} A) \bar{\chi}_j \chi_j \right) \\ & - \frac{1}{2} \left(f^*(A^*) \frac{\partial^2 f}{\partial A^2}(A) B^2 + h.c. \right) - \left| \frac{\partial f}{\partial A}(A) B \right|^2 \\ & + \left(\sum_{i,j} y_{ij} B \bar{\chi}_j \lambda_i + h.c. \right), \end{aligned} \quad (63)$$

where

$$f(A) \equiv \frac{\frac{1}{2} - \cos A}{2 - \cos A}. \quad (64)$$

Similarly to Section 3.2, $h_{Li}, h_{Rj} > 0$ and $M_{\lambda i}, M_{\chi j}$ and y_{ij} are real.

Then the mode functions of the zero-modes $\lambda_{Li,0}(x)$, $\chi_{Rj,0}(x)$ and $b_0(x)$ in λ_i , χ_j and B , which are localized on our wall, are

$$\begin{aligned} \varphi_{\lambda_{Li},0}(y) &= C_{\lambda_{Li}} e^{\int_0^y dy' (h_{Li} \text{Re} A_{\text{cl}}(y') + M_{\lambda i})}, \\ \varphi_{\chi_{Rj},0}(y) &= C_{\chi_{Rj}} e^{\int_0^y dy' (h_{Rj} \text{Re} A_{\text{cl}}(y') - M_{\chi j})}, \\ \phi_{B,0}(y) &= C_A \partial_y A_{\text{cl}}(y), \end{aligned} \quad (65)$$

where $C_{\lambda_{Li}}, C_{\chi_{Rj}}$ are complex and C_A is real.

Strictly speaking, we must check up whether the above functions are periodic because y is the coordinate of the extra dimension compactified on S^1 in this model. This condition is not satisfied unless $\int_0^{2\pi R} dy' (h_{Li} \text{Re} A_{\text{cl}}(y') + M_{\lambda i}) = \int_0^{2\pi R} dy' (h_{Rj} \text{Re} A_{\text{cl}}(y') - M_{\chi j}) = 0$. Thus there is no fermionic zero-mode in the strict meaning. Nevertheless, since there exist the zero-modes in a single-wall background[26], it is natural to suppose that “pseudo-zero-modes” exist when the distance between the walls is large enough. So we will assume

⁵ Strictly speaking, there is a lower bound for the wall distance.

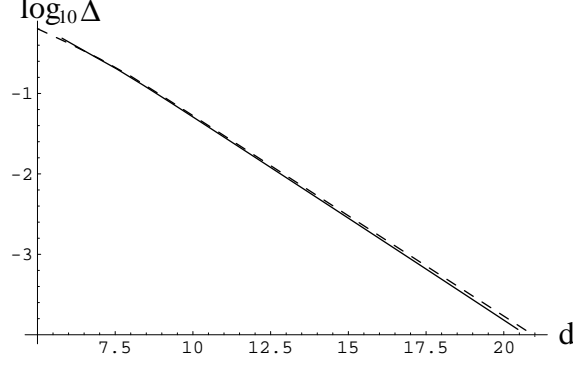


Figure 11: The measure of the CP violation Δ as a function of the wall distance in the case of Eq.(67). The solid line represents the result by using the exact configuration $A_{\text{cl}}(y)$ and the dashed line is the result by the approximation Eq.(68). The distance d is normalized by $1/(g\Lambda^{3/2})$.

that there exist the pseudo-zero-modes in λ_i and χ_j , and their mode functions are well approximated by Eq.(65) near our wall.

The effective Yukawa couplings $y_{\text{eff}ij}$ involving $\lambda_{L i,0}$, $\chi_{R j,0}$ and b_0 are calculated as

$$y_{\text{eff}ij} = y_{ij} \int_{-\pi R}^{\pi R} dy \phi_{B,0}(y) \varphi_{\chi_{R j,0}}(y) \varphi_{\lambda_{L i,0}}(y). \quad (66)$$

Then we can calculate the quantity Δ defined by Eq.(27), from these $y_{\text{eff}ij}$. It is shown by the solid line in Fig.11 in the case that⁶

$$\begin{aligned} \frac{1}{g^2 \Lambda^{1/2}}(h_{L1}, h_{L2}, h_{L3}) &= \frac{1}{g^2 \Lambda^{1/2}}(h_{R1}, h_{R2}, h_{R3}) = (20, 12, 8), \\ \frac{1}{g \Lambda^{3/2}}(M_{\lambda 1}, M_{\lambda 2}, M_{\lambda 3}) &= -\frac{1}{g \Lambda^{3/2}}(M_{\chi 1}, M_{\chi 2}, M_{\chi 3}) = (-16, 0, 6). \end{aligned} \quad (67)$$

As we can see from this plot, the CP violating effects decay exponentially as the wall distance increases.

Next we will confirm the validity of the approximation Eq.(35). We approximate $A_{\text{cl}}(y)$ near our wall by

$$A_{\text{cl}}(y) \simeq A_{\text{cl}}^{(1)}(y) + i \text{Im} A_{\text{cl}}^{(2)}(y - d) + i \text{Im} A_{\text{cl}}^{(2)}(y + d), \quad (68)$$

where $d = \pi R$ is the distance between the walls. Here note that there are both contributions of the other wall at $y = d$ and $y = -d$.

Namely, (pseudo-)zero-modes trapped on our wall can be approximated near our wall as follows.

$$\varphi_{\lambda_{L i,0}}(y) = C_{\lambda_{L i}} e^{\int_0^y dy' (h_{L i} \text{Re} A_{\text{cl}}^{(1)}(y') + M_{\lambda i})},$$

⁶ By considering the redefinition Eq.(52), we have restored the dependence of g and Λ .

$$\begin{aligned}
\varphi_{\chi_{Rj},0}(y) &= C_{\chi_{Rj}} e^{\int_0^y dy' (h_{Rj} \text{Re} A_{\text{cl}}^{(1)}(y') - M_{\chi j})}, \\
\phi_{B,0}(y) &= C_A \partial_y \left(A_{\text{cl}}^{(1)}(y) + i \text{Im} A_{\text{cl}}^{(2)}(y-d) + i \text{Im} A_{\text{cl}}^{(2)}(y+d) \right), \tag{69}
\end{aligned}$$

The measure of the CP violation Δ calculated by these approximate mode functions is plotted by the dashed line in Fig.11.

As shown in Fig.11, we can conclude that the approximation Eq.(68) is pretty good.

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